DAY NINETEEN

Current Electricity

Learning & Revision for the Day

Electric Current

Electric current is defined as the amount of charge flowing across any section of wire per unit time. If charge Δq passes through the area in time interval Δt at uniform rate, then current i is defined as

$$i = \frac{\Delta q}{\Delta t} \quad \ldots(\text{i})$$

SI unit of electric current is ampere (A).

- Conventional direction of flow of current is taken to be the direction of flow of positive charge or opposite to the direction of flow of negative charge.
- Electric current is a scalar as it does not follow the law vector of addition.

Current Density

Current per unit area is termed as current density.

$$J = \frac{I}{A} \,(\text{Am}^{-2})$$

It is a vector quantity.

Drift Velocity

- Drift velocity is the average uniform velocity acquired by conduction electrons inside a metallic conductor on application of an external electric field.

  The drift velocity is given by the relation

$$\mathbf{v}_d = -\frac{e\,\mathbf{E}}{m}\tau$$

  where, τ known as relaxation time.

- Drift velocity per unit electric field is called the **mobility** of the electrons. Thus, mobility,

$$\mu = \left|\frac{\mathbf{v}_d}{\mathbf{E}}\right| = \frac{e}{m}\tau$$

- In terms of drift speed, electric current flowing through a conductor is expressed as $I = nAev_d$

  where, A = cross-section area of conductor,

  n = number of conduction electrons per unit volume,

  v_d = drift velocity of electrons

  and e = charge of one electron.

Ohm's Law

Ohm's law states that the physical conditions such as temperature, mechanical strain, etc., are kept constant, then current (i) flowing through a conductor is directly proportional to the potential difference across its two ends.

i.e. $i \propto V$ or $V \propto i$ or $V = Ri$ or $\frac{V}{i} = R = a$ constant,

where R depends on the nature of material and it given dimension.

Electrical Resistivity

The resistance of a resistor (an element in a circuit with some resistance R) depends on its geometrical factors (length, cross-sectional area) as also on the nature of the substance of which the resistor is made. Electrical resistance of a rectangular slab depends on its length (l) and its cross-sectional area (A).

i.e., $R \propto l$ ...(i)

and $R \propto \frac{1}{A}$ ...(ii)

Combining the two relations, we get

$$R \propto \frac{l}{A} \quad \text{...(iii)}$$

or $R = \frac{\rho l}{A}$ ...(iv)

where, ρ is a constant of proportionality called resistivity.

$$\rho = \frac{m}{ne^2\tau}$$

Resistance of Different Materials

Resistance offered by the conductors is minimum while resistance offered by an insulator is maximum. Semiconductors have resistance which is intermediate to conductor and insulator.

Electrical Resistance

Electrical resistance is defined as the ratio in the potential difference (v) across the ends of the conductor to the current (i) flowing through it, i.e. $R = \frac{V}{i}$

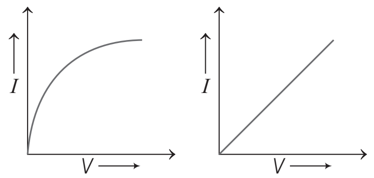
The SI unit of electrical restristance is Ω (ohm) and its dimension is $[ML^2T^{-3}A^{-2}]$.

Resistance of Different Materials

A perfect conductor would have zero resistivity and a perfect insulator would have infinite resistivity. Though these are ideal limits, the electrical resistivity of substances has a very wide range. Metals have low resistivity of 10^{-8} Ωm to 10^{-6} Ωm, while insulators like glass or rubber have resistivity, some 10^{18} times (or even more) greater, Generally, good electrical conductors like metals are also good conductors of heat, while insulators like ceramic or plastic materials are also poor thermal conductors.

V-I Characteristics of Ohmic and Non-ohmic Conductors

Substances obeying Ohm's law are called **Ohmic resistors**, e.g. metals and their alloys. Substances which do not obey Ohm's law are called **non-ohmic resistors**, e.g. electrolytes, gases, thermionic tubes, transistors, rectifiers, etc.



Non-ohmic resistors Ohmic resistors

Colour Code for Resistors

The value of resistance used in electrical and electronic circuits vary over a wide range. These resistances are usually carbon resistances and a colour code is used to indicate the value of resistance.

Their value ranges from kilo-ohm to mega-ohm. Their percentage accuracy is indicated by a colour code printed on them. Carbon resistors are compact, inexpensive and are used in electronic circuits.

Colour Code for Carbon Resistors

Colour	Letter of remember	Number	Multiplier	Tolerance
Black	B	0	10^0	—
Brown	B	1	10^1	—
Red	R	2	10^2	—
Orange	O	3	10^3	—
Yellow	Y	4	10^4	—
Green	G	5	10^5	—
Blue	B	6	10^6	—
Violet	V	7	10^7	—
Grey	G	8	10^8	—
White	W	9	10^9	—
Gold	—	—	10^{-1}	5%
Silver	—	—	10^{-2}	10%
No colour	—	—	—	20%

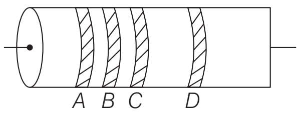
Now to find the colour coding of carbon resistor, we must remember the bold capital letters of the following sentences:

Black **B**rown **ROY** **G**reat **B**ritain **V**ery **G**ood **W**ife **W**earing **G**old **S**ilver **N**ecklace

Or

Black **B**rown **R**ods of **Y**our **G**ate **B**ecame **V**ery **G**ood **W**hen **G**iven **S**ilver **C**olour

The colours of first two bands A and B correspond to figures and the colour of the third band C represents multipliers, respectively. The fourth band represents the tolerance.



e.g. Consider a carbon resistor of bands A and B of black and red colour having figures 0 and 2. The third band C of green colour having multiplier 10^5.

$\therefore$ Resistance of the value is given by

$$R = 02 \times 10^5 \Omega$$

But the fourth band D having gold colour, which represents a tolerance of $\pm 5\%$.

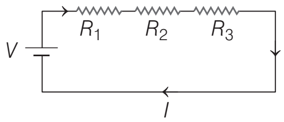
Hence, the value of carbon resistance is

$$R = 02 \times 10^5 \Omega \pm 5\%$$

Series and Parallel Combinations of Resistors

Series Combination

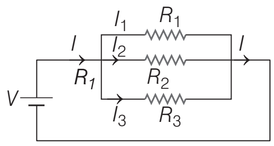
A series circuit is a circuit in which resistors are arranged in a chain, so the current has only one path to take. The current is the same through each resistor. The total resistance of the circuit is found by simply adding up the resistance values of the individual resistors. Equivalent resistance of resistors in series



$$R = R_1 + R_2 + R_3 + \dots$$

Parallel Combination

A parallel circuit is a circuit in which the resistors are arranged with their heads connected together and their tails connected together. The current in a parallel circuit breaks up, with some flowing along each parallel branch and recombining, when the branches meet again. The voltage across each resistor is parallel is the same.



The total resistance of a set of resistors in parallel is found by adding up the reciprocals of the resistance values, and then, taking the reciprocal of the total.

The equivalent resistance of resistors in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Temperature Dependence of Resistance

Resistance and resistivity of metallic conductors increases with increase in temperature. The relation is written as

$$R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2) \text{ and } \rho_\theta = \rho_0 (1 + \alpha\theta + \beta\theta^2)$$

where, R_0 and ρ_0 are values of resistance and resistivity at $0°C$ and R_θ and ρ_θ at $\theta°C$. α and β are two constants whose value vary from metal to metal.

Electric Energy and Power

Whenever the electric current is passed through a conductor, it becomes hot after short time. This effect is known as **heating effect** of current or **Joule heating effect.**

$$H = W = I^2 Rt \text{ joule} = \frac{I^2 Rt}{418} \text{ cal}$$

The rate at which work is done by the source of emf in maintaining the effect of current in a circuit is called electric power of the circuit,

$$P = VI \text{ watt}$$

Other expressions for power,

$$P = I^2 R \text{ watt} \Rightarrow P = \frac{V^2}{R}$$

Electric Cell

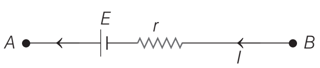
An electric cell is a device which maintains a continuous flow of charge (or electric current) in a circuit by a chemical reaction. In an electric cell, there are two rods of different metals called electrodes.

Internal Resistance of a Cell

Thus, when a current is drawn through a source, the potential difference between the terminal of the source is

$$V = E - ir$$

This can also be shown as below



$$V_A - E + Ir = V_B$$

or $$V_A - V_B = E - Ir$$

Following three special cases are possible

(i) If the current flows in opposite direction (as in case of charging of battery), then $V = E + Ir$

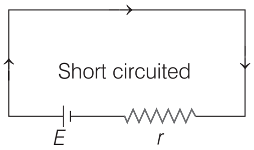
(ii) $V = E$, if the current through the cell is zero.

(iii) $V = 0$, if the cell is short circuited.

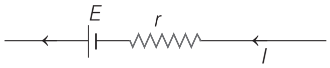
This is because current in the circuit,

$$I = \frac{E}{r} \quad \text{or} \quad E = Ir$$

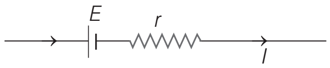
$$\therefore \qquad E - Ir = 0 \quad \text{or} \quad V = 0$$



Thus, we can summarise, it was follows

⇒ 

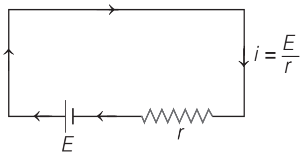
$$V = E - ir \text{ or } V < E$$

⇒ 

$$V = E + Ir \text{ or } V > E$$

⇒ 

$$V = E, \text{ if } I = 0$$



$V = 0$ is short circuited

Potential Difference and emf of a Cell

Electromotive force (emf) of a cell is the terminal potential difference of cell when it is in an **open circuit,** i.e. it is not supplying any current to the external circuit. However, when it is supplying a current to an external resistance, the voltage across the terminals of cell is called the **terminal voltage** or **terminal potential difference.**

If E be the emf and r the internal resistance of a cell and a resistance R is joined with it, then current in the circuit, $I = \frac{E}{R + r}$ and terminal potential difference,

$$V = IR = \frac{ER}{(R + r)}$$

or
$$V = E - Ir$$

Internal resistance of cell,

$$r = \left(\frac{E - V}{V}\right)R = R\left(\frac{E}{V} - 1\right)$$

Terminal voltage is more than emf of cell when cell is charged and it is given by $V = E + Ir$.

Combination of Cells in Series and in Parallel

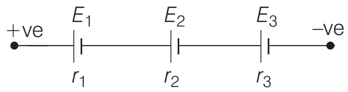
A group of cells is called a battery. Two common grouping of cells are

1. Series Grouping

In series grouping, if all the cells are joined so as to supply current in the same direction, then resultant emf,

$$E_{eq} = E_1 + E_2 + E_3 + \dots$$

However, if one or more cells are joined so as to supply current in reverse direction, then emf of that/those cells is taken as negative, while calculating the equivalent emf.



The equivalent internal resistance of the cell,

$$r_{eq} = r_1 + r_2 + r_3 + \dots$$

If n cells, each of emf E and internal resistance r, are joined in series, then

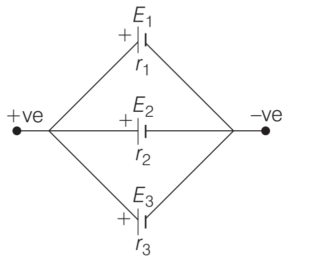
$$E_{eq} = nE \quad \text{and} \quad r_{eq} = nr$$

2. Parallel Grouping

In parallel grouping, if positive terminals of all cells have been joined at one point and all negative terminals at another point, then $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots$

The equivalent emf of the parallel grouping is given by

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \dots$$



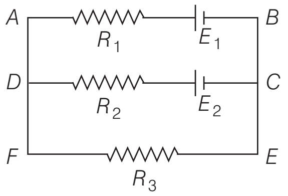
If n cells, each of emf E and internal resistance r, all joined in parallel, then

$$r_{eq} = \frac{r}{n}$$

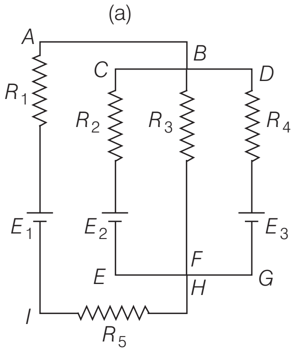
But
$$E_{eq} = E$$

Kirchhoff's Laws and their Applications

Many electric circuits cannot be reduced to simple series parallel combinations. For example, two circuits that cannot be so broken down are shown in figure



(a)



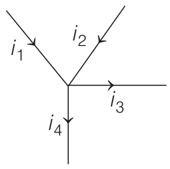
(b)

However, it is always possible to analyze such circuits by applying two rules, derived by Kirchhoff.

Junction Rule

The algebraic sum of the currents at any junction is zero.

i.e. $$\sum_{\text{junction}} i = 0$$



This law can also be written as, "the sum of all the currents directed towards a point in circuit is equal to the sum of all the currents directed away from that point."

Thus, in figure, $i_1 + i_2 = i_3 + i_4$

The junction rule is bases on **conservation of electric charge**.

Loop Rule

The algebraic sum of the potential difference in any loop including those associated emf's and those of resistive elements, must be equal to zero.

That is, $$\sum_{\text{closed loop}} \Delta V = 0$$

This law represent **conservation of energy**.

Applying Kirchhoff's law for the following circuit, we have Resulting equation is

$$V_{r_1} + V_{r_2} + V_{r_3} - 10 = 0.$$

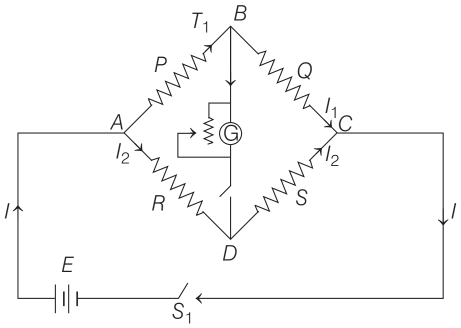
Wheatstone's Bridge

For measuring accurately any resistance Wheatstone bridge is widely used. It is an arrangement of four resistances used to measure one of them in terms of the other three.

Consider four resistances P, Q, R and S are connected in the four arms of a quadrilateral according to figure, the bridge is said to be balanced when galvanometer gives zero deflection, i.e. potential at point B and D is same ($V_B = V_D$).

In this condition,

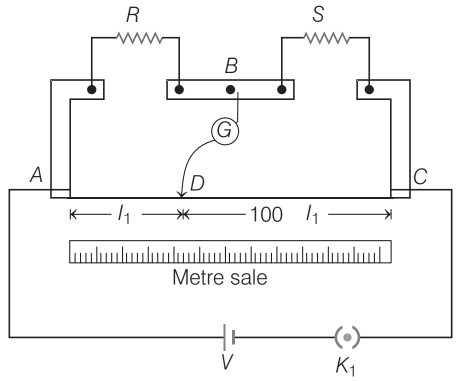
$$\frac{P}{Q} = \frac{R}{S}$$



Meter Bridge (Special Case of Wheatstone Bridge)

This is the simplest form of Wheastone bridge and is specially useful for comparing resistances more accurately. The construction of the meter bridge is shown in the figure. It consists of one metre resistance wire clamped between two metallic strips bent at right angles and it has two points for connection.

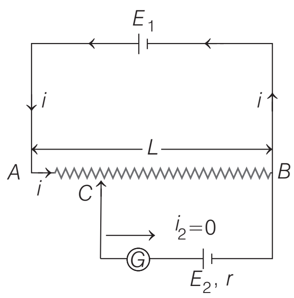
There are two gaps; in one of whose value is to be determined is connected. The galvanometer is connected with the help of jockey across BD and the cell is connected across AC. After making connections, the jockey is moved along the wire and the null point is found. Wheatstone bridge, wire used is of uniform material and cross-section. the resistance can be found with the help of the following relation



$$\frac{R}{S} = \frac{l_1}{(100 - l_1)} \quad \text{or} \quad R = S\,\frac{l_1}{100 - l_1}$$

Potentiometer

Principle Potentiometer is an ideal device to measure the potential difference between two points. It consists of a long resistance wire AB of uniform cross-section in which a steady direct current is set up by means of a battery.



Potential gradient,

$$k = \frac{\text{Potential difference across } AB}{\text{Total length}}$$

$$= \frac{V_{AB}}{L}$$

$$= \frac{iR_{AB}}{L} = i\lambda$$

where, $\lambda = \frac{R_{AB}}{L}$ = resistance per unit length of potentiometer wire.

The emf of source balanced between points B and C

$$E_2 = kl$$

$$= i\frac{R_{AB}}{l} \times l = iR_{CB}$$

Here, AB is a long uniform resistance wire (length AB may be ranging from 1 m to 10 m). E_0 is a battery whose emf is known supplying a constant current I for flow through the potentiometer wire. If R be the total resistance of potentiometer wire and L its total length, then potential gradient, i.e. fall in potential per unit length along the potentiometer will be

$$k = \frac{V}{L} = \frac{IR}{L}$$

$$= \frac{E_0 R}{(R_0 + R)\,L}$$

where, E_0 = emf of battery,

R_0 = resistance inserted by means of rheostat R_h

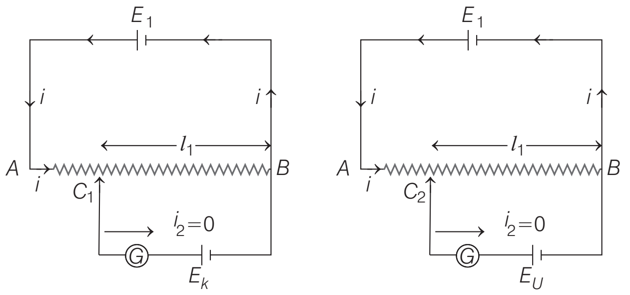
k = potential gradient.

$L \rightarrow$ balancing length

$J \rightarrow$ jockey.

Applications of Potentiometer

(i) To find emf of an unknown battery



We calibrate the device by replacing E_2 by a source of known emf E_k and then by unknown emf E_u. Let the null points are obtained at lengths l_1 and l_2.

Then, $E_K = i(\rho l_1)$ and $E_U = i\,(\rho l_2)$

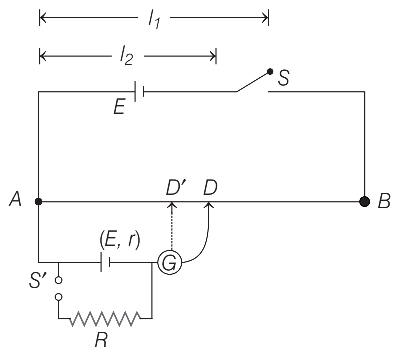
Here, ρ = resistance of wire AB per unit length

$$\therefore \quad \frac{E_K}{E_U} = \frac{l_1}{l_2} \text{ or } E_U = \left(\frac{l_2}{l_1}\right) E_K$$

So, by measuring the lengths l_1 and l_2, we can find the emf of an unknown battery.

(ii) To find the internal resistance of a cell

Firstly, the emf E of the cell is balanced against a length $AD = l_1$. For this, the switch S' is left opened and S is closed. A known resistance R is then connected to the cell as shown. The terminal voltage V is now balanced against a smaller length $AD' = l_2$. Here, now switch is opened and S' is closed.



Then, $$\frac{E}{V} = \frac{l_1}{l_2}$$

Since, $$\frac{E}{V} = \frac{R + r}{R} \qquad \{\because E = i(R + r) \text{ and } V = iR\}$$

or $$\frac{R + r}{R} = \frac{l_1}{l_2} \Rightarrow r = \left(\frac{l_1}{l_2} - 1\right) R$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A wire of length L and 3 identical cells of negligible internal resistance are connected in series.Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

(a) 4 (b) 6 (c) 8 (d) 9

2 In a large building, there are 15 bulbs of 40W, 5 bulbs of 100W, 5 fans of 80 W and 1 heater of 1kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse is **→ JEE Main 2014**

(a) 8A (b)10A (c) 12A (d) 14A

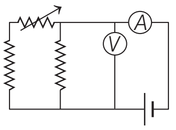
3 Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse? **→ AIEEE 2012**

(a) Both (b) 100 W (c) 25 W (d) Neither

4 A given resistor has the following colour scheme of the various strips on it brown, black, green and silver. Its value in ohm is

(a) $1.0 \times 10^4 \pm 10\%$ (b) $1.0 \times 10^5 \pm 10\%$
(c) $1.0 \times 10^6 \pm 10\%$ (d) $1.0 \times 10^7 \pm 10\%$

5 Correct set up to verify Ohm's law is **→ JEE Main (Online) 2013**

(a)


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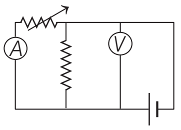

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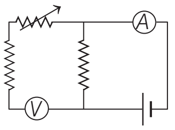

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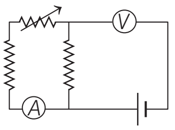
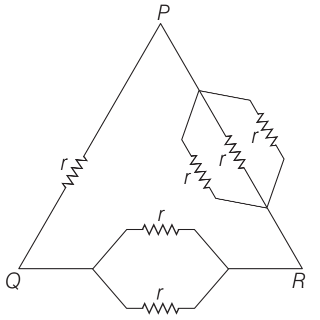
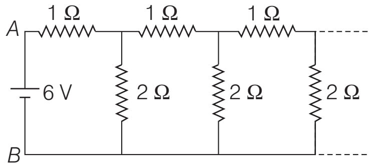


6 Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between **→ JEE Main (Online) 2013**



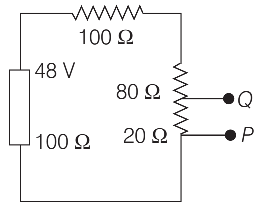
(a) P and R (b) P and Q
(c) Q and R (d) Any two points

7 An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistance as shown in figure. The 6 V battery between A and B has negligible internal resistance, then effective resistance between A and B is



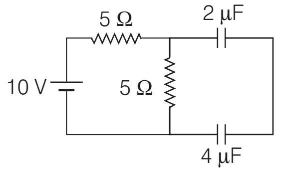
(a) 3 Ω (b) 2 Ω
(c) 6/5 Ω (d) 5/6 Ω

8 In the circuit in the figure below, the potential difference across P and Q will be nearest to



(a) 9.6 V (b) 6.6 V
(c) 4 V (d) 3.2 V

9 The charge on the 4 μF capacitor in the steady state, is

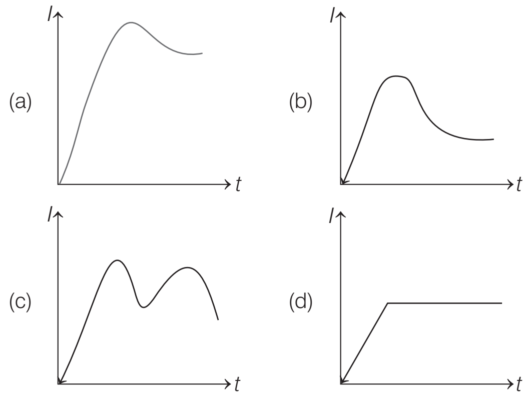


(a) $\frac{20}{3}$ μC (b) $\frac{10}{3}$ μC
(c) $\frac{5}{6}$ μC (d) 10μC

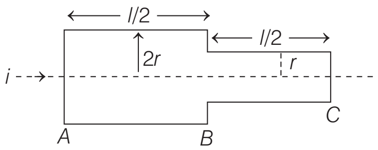
10 The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by **→ JEE Main 2016 (Offline)**

(a) linear increase for Cu, linear increase for Si
(b) linear increase for Cu, exponential increase for Si
(c) linear increase for Cu, exponential decrease for Si
(d) linear decrease for Cu, linear decrease for Si

11 When an electric heater is switched ON, the current flowing through it is plotted against time (t). Taking into account the variation of resistance with temperature, which of the following best represents the variations

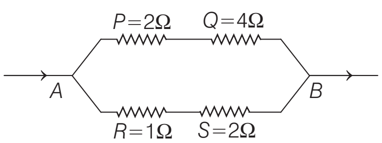


12 Two bars of radii r and $2r$ are kept in contact as shown. An electric current i is passed through the bars. Which one of the following is correct?



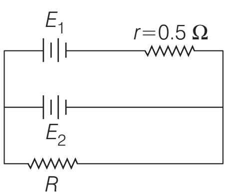
(a) Heat produced in bar BC is 4 times the heat produced in bar AB
(b) Electric field in both halves is equal
(c) Current density across AB is double that of across BC
(d) Potential difference across AB is 4 times that of across BC

13 Which of the four resistances P, Q, R and S generate the greatest amount of heat when a current flows from A to B? **➞ JEE Main (Online) 2013**



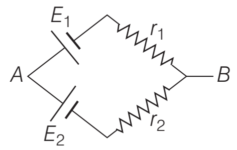
(a) Q (b) S (c) P (d) R

14 A DC source of emf $E_1 = 100$ V and internal resistance $r = 0.5\ \Omega$, a storage battery of emf $E_2 = 90$ V and an external resistance R are connected as shown in figure. For what value of R no current will pass through the battery? **➞ JEE Main (Online) 2013**



(a) $5.5\ \Omega$ (b) $3.5\ \Omega$
(c) $4.5\ \Omega$ (d) $2.5\ \Omega$

15 Two batteries of emf E_1 and $E_2 (E_2 > E_1)$ and internal resistances r_1 and r_2, respectively are connected in parallel as shown in figure.



(a) Two equivalent emf E_{eq} of the two cells is between E_1 and E_2, i.e. $E_1 < E_{eq} < E_2$
(b) The equivalent emf E_{eq} is smaller than E_1
(c) The E_{eq} is given by $E_{eq} = E_1 + E_2$ always
(d) E_{eq} is independent of internal resistance r_1 and r_2

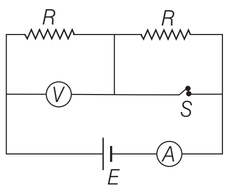
16 Match the following columns.

Column I		Column II	
A.	Smaller the resistance greater the current	1.	If the same voltage is applied
B.	Greater the resistance smaller the current	2.	If the same current is passed
C.	Greater the resistance smaller the power	3.	When resistances are connected in series
D.	Greater the resistance greater the power	4.	When resistances are connected in parallel

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	4	1	2	3
(c)	4	3	1	2	(d)	3	4	2	1

17 In the circuit shown, battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown in figure. When switch S is opened, match the parameter of column I with the effects in column II.

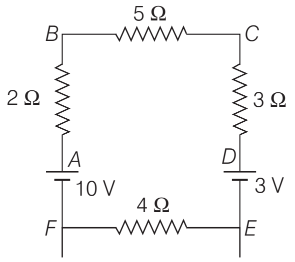


Column I		Column II	
A.	Equivalent resistance across the battery	1.	Remains same
B.	Power dissipated by left resistance R	2.	Increases
C.	Voltmeter reading	3.	Decreases
D.	Ammeter reading	4.	Becomes zero

Codes

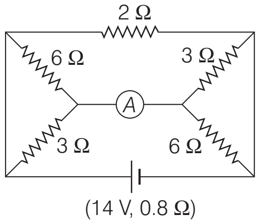
	A	B	C	D		A	B	C	D
(a)	2	3	3	3	(b)	3	2	2	2
(c)	4	1	1	1	(d)	1	4	4	4

18 In the circuit shown in figure. The point F is grounded. Which of the following is wrong statement?



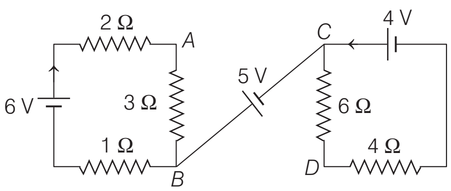
(a) D is at 5 V
(b) E is at zero potential
(c) The current in the circuit will be 0.5 A
(d) None of the above

19 The reading of ammeter as shown in figure is,



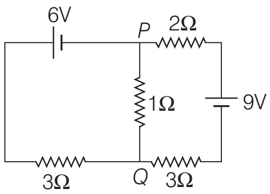
(a) 6.56 A (b) 3.28 A
(c) 2.18 A (d) 1.09 A

20 What is the potential difference between points A and D of circuit as shown in figure?



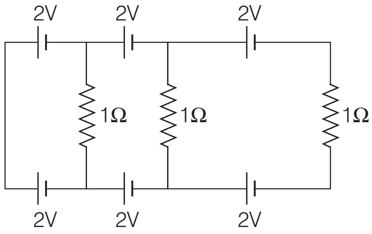
(a) 5 V (b) 9 V
(c) 10.4 V (d) 11.4 V

21 In the circuit shown below, the current in the 1 Ω resistor is **→ JEE Main 2014**



(a) 1.3A, from P to Q
(b) 0A
(c) 0.13 A, from Q to P
(d) 0.13 A, from P to Q

22 In the below circuit, the current in each resistance is **→ JEE Main 2017 (Offline)**



(a) 0.25 A (b) 0.5 A (c) 0 A (d) 1 A

23 Which of the following statements is false? **→ JEE Main 2017 (Offline)**

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed
(b) A rheostat can be used as a potential divider
(c) Kirchhoff's second law represents energy conservation
(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

24 In a metre bridge experiment, null point is obtained at 40 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then the new position of the null points from the same end, if one decides to balance a resistance of $3X$ against Y, will be close to **→ JEE Main (Online) 2013**

(a) 80 cm (b) 75 cm (c) 67 cm (d) 50 cm

25 On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances? **→ JEE Main 2018**

(a) 990 Ω (b) 505 Ω (c) 550 Ω (d) 910 Ω

26 In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 4 Ω resistance; and is at a length of 3 m when the cell is shunted by a 8 Ω resistance. The internal resistance of the cell is, then

(a) 12 Ω (b) 8 Ω (c) 16 Ω (d) 1 Ω

27 A 6 V battery is connected to the terminals of a 3 m long wire of uniform thickness and resistance of 100 Ω. The difference of potential between two points on the wire separated by a distance of 50 cm will be

(a) 2 V (b) 3 V (c) 1 V (d) 15 V

28 The current in the primary circuit of potentiometer is 0.2A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} Ω m and 8×10^{-7} m^2 respectively. The potential gradient will be equal to **→ AIEEE 2011**

(a) 0.2 V/m (b) 1 V/m (c) 0.3 V/m (d) 0.1 V/m

Direction (Q. Nos. 29-32) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these question also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

29 **Statement I** As temperature decreases, the relaxation time of a conducting material increases.

Statement II Number of collisions per unit time of electrons with lattice ions increases as the temperature increases.

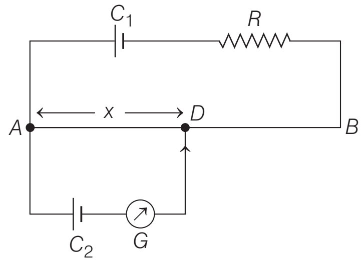
30 **Statement I** Potential difference across the terminals of a battery can be greater than its emf.

Statement II When current is taken from battery, $V = E - Ir$ (Symbols have their usual meaning).

31 **Statement I** In a meter bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement II Resistance of a metal increases with increase in temperature.

32 **Statement I** In the potentiometer circuit shown in figure, E_1 and E_2 are the emfs of cells C_1 and C_2 respectively with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.



Statement II At the balance point, the potential difference between AD due to cell $C_1 = E_2$, the emf of cell C_2.

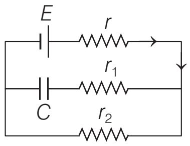
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 If a wire is stretched to make it 0.1% longer, its resistance will **→ AIEEE 2012**

(a) increase by 0.2% (b) decrease by 0.2%
(c) decrease by 0.05 % (d) increase by 0.05%

2 In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



→ JEE Main 2017 (Offline)]

(a) $CE \dfrac{r_1}{(r_2 + r)}$ (b) $CE \dfrac{r_2}{(r + r_2)}$
(c) $CE \dfrac{r_1}{(r_1 + r)}$ (d) CE

3 When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5 \times 10^{-4}\ \text{ms}^{-1}$. If the electron density in the wire is $8 \times 10^{28}\text{m}^{-3}$ the resistivity of the material is close to **→ JEE Main 2015**

(a) $1.6 \times 10^{-8}\ \Omega\text{m}$ (b) $1.6 \times 10^{-7}\ \Omega\text{m}$
(c) $1.6 \times 10^{-6}\ \Omega\text{m}$ (d) $1.6 \times 10^{-5}\ \Omega\text{m}$

4 A letter A is constructed of a uniform wire with resistance 1.0 Ω per cm. The sides of the letter are 20 cm and the cross piece in the middle is 10 cm long. The apex angle is 60°. The resistance between the ends of the legs is close to **→ JEE Main (Online) 2013**

(a) 50.0 Ω (b) 10 Ω
(c) 36.7 Ω (d) 26.7 Ω

5 Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between **→ JEE Main 2018**

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V
(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

6 In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **→ JEE Main 2018**

(a) 1 Ω (b) 1.5 Ω (c) 2 Ω (d) 2.5 Ω

7 Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **→ AIEEE 2010**

(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$ (b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

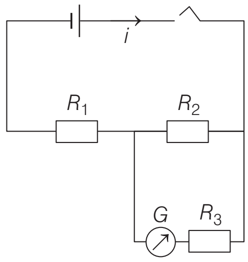
(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

8 There are two concentric spheres of radius a and b respectively. If the space between them is filled with medium of resistivity ρ, then the resistance of the intergap between the two spheres will be

(a) $\frac{\rho}{4\pi(b+a)}$ (b) $\frac{\rho}{4\pi}\left(\frac{1}{b}-\frac{1}{a}\right)$

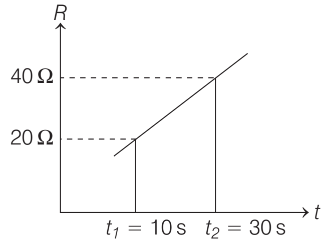
(c) $\frac{\rho}{4\pi}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$ (d) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

9 To find the resistance of a galvanometer by the half deflection method the following circuit is used with resistances $R_1 = 9970\ \Omega$, $R_2 = 30\ \Omega$ and $R_3 = 0$. The deflection in the galvanometer is d. With $R_3 = 107\ \Omega$ the deflection changed to $\frac{d}{2}$. The galvanometer resistance is approximately **→ JEE Main (Online) 2013**



(a) 107 Ω (b) 137 Ω (c) 107/2 Ω (d) 77 Ω

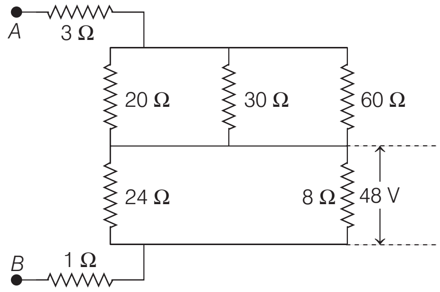
10 A source of emf $E = 10$ V and having negligible internal resistance is connected to a variable resistance. The resistance varies as shown in figure. The total charge that has passed through the resistor R during the time interval from t_1 to t_2 is



(a) $40 \log_e 4$ (b) $30 \log_e 3$

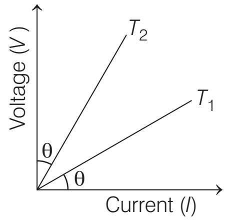
(c) $20 \log_e 2$ (d) $10 \log_e 2$

11 The potential difference across 8Ω resistance is 48V as shown in figure. The value of potential difference across points A and B will be



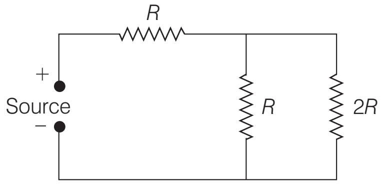
(a) 62 V (b) 80 V (c) 128 V (d) 160 V

12 The V-I graph for a conductor at temperatures T_1 and T_2 are as shown in the figure, The term $T_2 - T_1$ is proportional to



(a) $\cos 2\theta$ (b) $\sin 2\theta$ (c) $\cot 2\theta$ (d) $\tan 2\theta$

13 The charge supplied by source varies with time t as $Q = at - bt^2$. The total heat produced in resistor $2R$ is



(a) $\frac{a^3R}{6b}$ (b) $\frac{a^3R}{27b}$

(c) $\frac{a^3R}{3b}$ (d) None of these

ANSWERS

SESSION 1	**1** (b)	**2** (c)	**3** (c)	**4** (c)	**5** (a)	**6** (b)	**7** (b)	**8** (d)	**9** (a)	**10** (c)
	11 (b)	**12** (a)	**13** (b)	**14** (c)	**15** (a)	**16** (a)	**17** (a)	**18** (b)	**19** (c)	**20** (c)
	21 (c)	**22** (c)	**23** (a)	**24** (c)	**25** (c)	**26** (b)	**27** (c)	**28** (d)	**29** (a)	**30** (b)
	31 (d)	**32** (d)								
SESSION 2	**1** (a)	**2** (b)	**3** (d)	**4** (d)	**5** (b)	**6** (b)	**7** (d)	**8** (d)	**9** (d)	**10** (d)
	11 (d)	**12** (c)	**13** (b)							

Hints and Explanations

SESSION 1

1 Here, $R = \frac{\rho L}{A}; R_1 = \frac{\rho 2L}{A} = 2R$

As, $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

i.e. $d = \frac{m}{AL}$

$\therefore \quad m = ALd; m_1 = A2Ld = 2m$

Now, heat produced in first case

$$H_1 = \frac{(3E)^2}{R} \times t = ms\Delta T \quad \text{...(i)}$$

In the second case,

$$H_2 = \frac{(NE)^2}{2R} \times t = 2ms\Delta T \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$N = 6$

2 Total power (P) consumed

$= (15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000)$

$= 2500 \text{ W}$

As we know that,

Power, i.e. $P = VI$

$\Rightarrow \quad I = \frac{2500}{220} \text{A} = \frac{125}{11}$

$= 11.3 \text{ A}$

Minimum capacity should be 12 A.

3 Resistance of bulb is given by $R = \frac{V^2}{P}$.

As the rated power of bulb 25 W is less than 100 W, it implies that 25 W bulb has higher resistance. As in series combination, current through both the bulbs is same, so heating in 25 W bulb is more than that of 100 W bulb. So, 25 W bulb will get fused.

4 Numbers attached for brown, black, green and silver are 1, 0, 5, ± 10 %. Therefore, the resistance of given resistor

$= 10 \times 10^5 \ \Omega \pm 10\%$

$= 1.0 \times 10^6 \ \Omega \pm 10\%$

5 Ohm's law states that the current (I) flowing through a conductor is directly proportional to the potential difference (V) across its ends provided its physical conditions such as temperature, mechanical strain, etc. kept constant,

i.e. $\quad I \propto V$ or $V \propto I$

or $\quad V = RI \quad$ (where, R is constant)

Thus, in order to study Ohm's law experimentally, voltmeter (V) should be connected parallel to the resistor. However, ammeter (A) should be connected in series with the resistor.

6 By solving this, we get net resistance as,

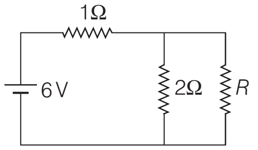
$$R_{PQ} = \frac{5}{11} r, R_{QR} = \frac{4}{11} r$$

and $\quad R_{PR} = \frac{3}{11} r$

$$R_{PQ} > R_{QR} > R_{PR}$$

Therefore, R_{PQ} is maximum.

7 Let R be the resistance of infinite ladder. The addition or subtraction of one step in the ladder will not affect the total resistance of network. Therefore, equivalent circuit will be as shown in figure.



Total resistance $= 1 + \frac{2 \times R}{R + 2} = R$

or $\quad R + 2 + 2R = R^2 + 2R$

$\Rightarrow \quad R^2 - R - 2 = 0$

On solving, we get

$R = 2 \ \Omega$

8 Total resistance of circuit

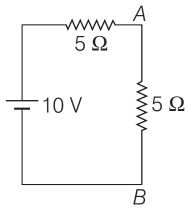
$= 100 + 100 + 80 + 20 = 300 \Omega$

Current $I = \frac{48}{300} = 0.16 \text{ A}$

Potential difference across P

and $Q = 20 \times 0.16 = 3.2 \text{ V}$

9 In the steady state, the capacitors are fully charged and acts as open circuit, so the equivalent circuit in steady state would be as shown alongside figure.



Steady state current $I = \frac{10}{5 + 5} = 1 \text{ A}$

So, potential drop across AB is

$V = 5 \text{V}$

Sum of potential difference across $2\mu\text{F}$ and $4\,\mu\text{F}$ capacitors is 5 V. As capacitors are in series, charges on them would be same, let us say it is q.

From KVL, $\frac{q}{2} + \frac{q}{4} = 5$

$\Rightarrow \quad q = \frac{20}{3} \mu\text{C}$

10 As, we know copper is a conductor, so increase in temperature, increases the resistance. Then, silicon (Si) is semiconductor, so with increase in temperature, resistance will decrease.

11 The filament of the heater reaches its steady resistance when the heater reaches the steady temperature, which is much higher than room temperature. The resistance at room temperature is then much lower than the resistance of its steady state.

When the heater is switched ON, it draws a larger current than its steady state current as the filament heats up, its resistance increases and current falls to steady state value.

12 Current flowing through both the bars is equal. Now, the heat produced is given by

$H = I^2 Rt$

or $\quad H \propto R$

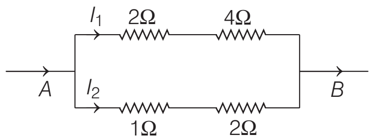
or $\quad \frac{H_{AB}}{H_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{(1/2r)^2}{(1/r)^2}$

$\left(\because R \propto \frac{1}{A} \propto \frac{1}{r^2}\right)$

$= \frac{1}{4}$

or $\quad H_{BC} = 4H_{AB}$

13 We know that, $I \propto \frac{1}{R}$,



Here, $\quad I_2 = \frac{6}{3 + 6} I = \frac{2}{3} I$

or $\quad I_1 = \frac{3}{6 + 3} I = \frac{1}{3} I$

Power rate in 2Ω of upper series

$= 2 \times \left(\frac{1}{3} I\right)^2 = \frac{2}{9} I^2$

Power rate in 4Ω of upper series

$= 4 \times \left(\frac{1}{3} I\right)^2 = \frac{4}{9} I^2$

Power rate of $1 \ \Omega$ in lower series

$= 1 \times \left(\frac{2}{3} I\right)^2 = \frac{4}{9} I^2$

Power rate of 2Ω in lower series

$= 2 \times \left(\frac{2}{3} I\right)^2 = \frac{8}{9} I^2$

$\therefore$ Greatest amount of heat is generated by S.

14 Given, $E_1 = 100\,V$,

$r = 0.5\,\Omega$,

$E_2 = 90\,V$

External resistance $= R$

For no current pass through the battery

$$\frac{100}{R + r} = \frac{90}{R}$$

$$\Rightarrow \quad \frac{10}{R + \frac{1}{2}\,\Omega} = \frac{9}{R}$$

$$\Rightarrow \quad 10R = 9R + 4.5\,\Omega$$

$$\therefore \quad R = 4.5\,\Omega$$

15 The equivalent emf of this combination is given by

$$E_{eq} = \frac{E_2 r_1 + E_1 r_2}{r_1 + r_2}$$

This suggest that the equivalent emf E_{eq} of the two cells is given by $E_1 < E_{eq} < E_2$.

16 Ohm's law, $I = \frac{V}{R}$

and power $= I^2 R = \frac{V^2}{R}$

When the resistors are connected in series, the effective resistance is more than that as when they are connected in parallel.

17 When switch S is opened, then right side resistance R which was short circuited earlier contributes to equivalent resistance. Hence, equivalent resistance across the battery increases, power dissipated by left resistance R decreases, voltmeter reading decreases and ammeter reading decreases.

18 Effective emf of circuit $= 10 - 3 = 7\,V$

Total resistance of circuit

$= 2 + 5 + 3 + 4$

$= 14\,\Omega$

Current, $I = 7/14 = 0.5\,A$

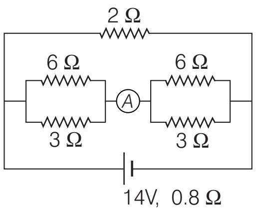
Potential difference between A and $D = 0.5 \times 10 = 5\,V$

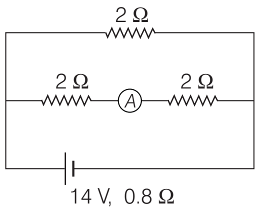
Potential at $D = 10 - 5 = 5\,V$

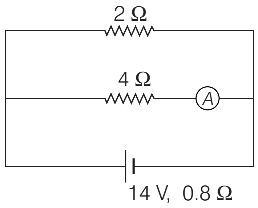
Potential at $E = 5 - 3 = 2\,V$

Hence, E cannot be at zero potential, as there is potential drop at E.

19 The equivalent circuit of the given circuit will be reduced to as shown in figure.







Total resistance of the circuit

$$= \frac{2 \times 4}{2 + 4} + 0.8 = \frac{8}{6} + 0.8 = \frac{12.8}{6}\,\Omega$$

Main current in the circuit

$$= \frac{14}{(12.8\ /\ 6)} = \frac{84}{12.8}\,A$$

Reading of ammeter

$$= \frac{84}{12.8} \times \frac{2}{6} = 2.18\,A$$

20 Let I_1 and I_2 be the currents drawn from cells of emf 6 V and 4 V in the circuits, respectively. Then,

$$I_1 = \frac{6}{2 + 3 + 1} = 1\,A$$

and $$I_2 = \frac{4}{6 + 4} = 0.4\,A$$

$V_A - V_B = 1 \times 3 = 3\,V$;

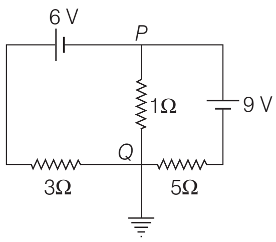
$V_B - V_C = 5\,V$

and $V_C - V_D = 0.4 \times 6 = 2.4\,V$

$\therefore \quad V_A - V_D = (V_A - V_B) + (V_B - V_C) + (V_C - V_D)$

$= 3 + 5 + 2.4 = 10.4\,V$

21 Connect point Q to ground and apply KCL. Consider the grounded circuit as shown below.



Applying KCL at point Q, we can write

Incoming current at $Q =$ outgoing current from Q

$$\Rightarrow \quad \frac{V + 6}{3} + \frac{V}{1} = \frac{9 - V}{5}$$

or $$V\left[\frac{1}{3} + \frac{1}{5} + 1\right] = \frac{9}{5} - 2$$

or $$V\left[\frac{5 + 3 + 15}{15}\right] = \frac{9 - 10}{5}$$

or $$V\left[\frac{23}{15}\right] = \frac{-1}{5}$$

or $$V = \frac{-3}{23} = -0.13\,V$$

$\therefore$ Current $= -1 \times 0.13$

$= -0.13\,A$

22 Each resistance is connected with two cells combined in opposite direction, so potential drop across each resistor is zero. Hence, the current through each of resistor is zero.

23 In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

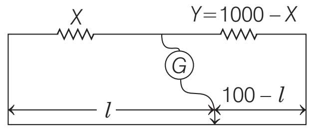
24 As, $\frac{x}{y} = \frac{40}{100 - 40} = \frac{40}{60}$

$$\frac{3x}{y} = 3\left(\frac{40}{60}\right) = \frac{120}{60} = \frac{2}{1}$$

Now, the total length $= 100$

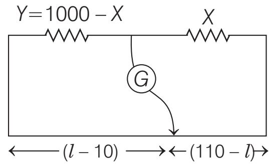
$\therefore$ Required length $= \frac{100}{3} \times 2 = 67\,cm$

25 We have, $X + Y = 1000\,\Omega$



Initially, $$\frac{X}{l} = \frac{1000 - X}{100 - l} \quad \ldots(i)$$

When X and Y are interchanged, then



$$\frac{1000 - X}{l - 10} = \frac{X}{100 - (l - 10)}$$

or $$\frac{1000 - X}{l - 10} = \frac{X}{110 - l} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{100 - l}{l} = \frac{l - 10}{110 - l}$$

$$(100 - l)(110 - l) = (l - 10)l$$

$$11000 - 100l - 110l + l^2 = l^2 - 10l$$

$$\Rightarrow \quad 11000 = 200l$$

$$\therefore \quad l = 55\,cm$$

Substituting the value of l in Eq. (i), we get

$$\frac{X}{55} = \frac{1000 - X}{100 - 55}$$

$$\Rightarrow \quad 20X = 11000$$

$$\therefore \quad X = 550\,\Omega$$

26 As, $\left(\frac{l-2}{2}\right)4 = \left(\frac{l-3}{3}\right)8$

$\Rightarrow \quad l = 6$

Therefore, $r = \left(\frac{l-2}{2}\right)4 = 8\,\Omega$

27 Potential gradient along the wire,

$K = \frac{6}{300}$ V/cm

Potential difference across 50 cm length is

$$V = k \times 50 = \frac{6}{300} \times 50 = 1\text{ V}$$

28 Potential gradient of a potentiometer,

$$K = \frac{I\rho}{A} = \frac{0.2 \times 4 \times 10^{-7}}{8 \times 10^{-7}} = 0.1\text{ V/m}$$

30 When the battery is undergoing charging processes, then

$$V = E + Ir > E$$

So, Statement I is correct.

Statement II is also correct but not explaining Statement I.

31 With increase in temperature, the value of unknown resistance will increase.

In balanced Wheatstone bridge condition,

$$\frac{R}{X} = \frac{l_1}{l_2}$$

Here,

R = value of standard resistance,

X = value of unknown resistance.

To take null point at same point or $\frac{l_1}{l_2}$ to remain unchanged, $\frac{R}{X}$ should also remain unchanged.

Therefore, if X is increasing R, should also increase.

32 If the diameter of wire AB is increased, its resistance will decrease. Hence, the potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.

SESSION 2

1 $R = \frac{\rho l}{A} = \frac{\rho l^2}{V}$ (V = volume)

$\therefore \quad \frac{\Delta R}{R} = 2\frac{\Delta l}{l} = +\ 0.2\%$

2 In steady state, no current flows through the capacitor. So, resistance r_1 becomes ineffective.

So, the current in circuit,

$$I = \frac{E}{r + r_2 \text{ (Total Resistance)}}$$

$\because$ Potential drop across capacitor

$= $ Potential drop across r_2

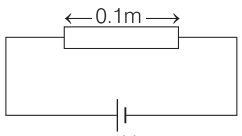
$= Ir_2 = \frac{Er_2}{r + r_2}$

$\therefore$ Stored charge of capacitor,

$Q = CV$

$= CE\frac{r_2}{r + r_2}$

3 According to the question,



$v_d = 2.5 \times 10^{-4}$ m/s

$\Rightarrow \quad n = 8 \times 10^{28}$ / m^3

We know that,

$J = nev_d$

or $\quad I = n_e v_d A$

where, symbols have their usual meaning.

$\Rightarrow \quad \frac{V}{R} = nev_d A$

or $\quad \frac{VA}{\rho L} = nev_d A$

or $\quad \frac{V}{\rho L} = nev_d$

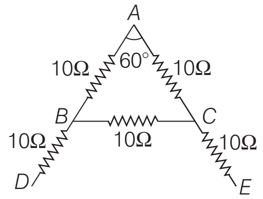
or $\quad \rho = \frac{V}{nev_d L}$

$$= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$\rho = 1.6 \times 10^{-5}\ \Omega$ m

4 We have, equivalent resistance in series

$R_1 + R_2 = 10 + 10 = 20\,\Omega$



and in parallel $= \frac{1}{R_P} = \frac{1}{20} + \frac{1}{10} = \frac{3}{20}$

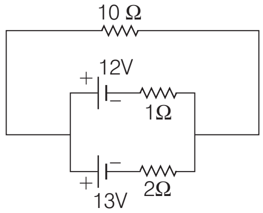
$\Rightarrow \quad R_P = \frac{20}{3}$

Therefore, $R_{eq} = \frac{20}{3} + 10 + 10$

$= \frac{20 + 30 + 30}{3}$

$= \frac{80}{3} = 26.66 = 26.7\,\Omega$

5



For parallel combination of cells,

$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\therefore \quad E_{eq} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}\text{ V}$$

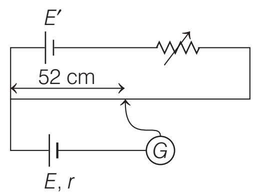
Potential drop across 10 Ω resistance,

$$V = \left(\frac{E}{R_{\text{total}}}\right) \times 10 = \frac{37/3}{\left(10 + \frac{2}{3}\right)} \times 10$$

$= 11.56$ V

$\therefore \quad V = 11.56$ V

6 With only the cell,

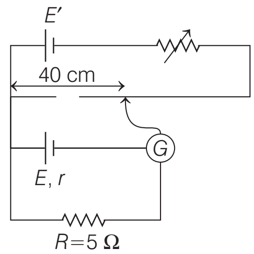


On balancing,

$E = 52 \times x$ ...(i)

where, x is the potential gradient of the wire.

When the cell is shunted,



Similarly, on balancing,

$V = E - \frac{Er}{(R + r)} = 40 \times x$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$\frac{E}{V} = \frac{1}{1 - \frac{r}{R + r}} = \frac{52}{40}$$

$\Rightarrow \quad \frac{E}{V} = \frac{R + r}{R} = \frac{52}{40}$

$\Rightarrow \quad \frac{5 + r}{5} = \frac{52}{40}$

$\Rightarrow \quad r = \frac{3}{2}\,\Omega \Rightarrow r = 1.5\,\Omega$

7 Let R_0 be the initial resistance of both conductors

∴ At temperature θ their resistances will be,

$$R_1 = R_0(1+\alpha_1\theta)$$

and $$R_2 = R_0(1+\alpha_2\theta)$$

For series combination,

$$R_s = R_1 + R_2$$

$$R_{s0}(1+\alpha_s\theta) = R_0(1+\alpha_1\theta) + R_0(1+\alpha_2\theta)$$

where, $R_{s0} = R_0 + R_0 = 2R_0$

∴ $2R_0(1+\alpha_s\theta) = 2R_0 + R_0\theta(\alpha_1+\alpha_2)$

or $$\alpha_s = \frac{\alpha_1+\alpha_2}{2}$$

For parallel combination,

$$R_{p0}(1+\alpha_p\theta) = \frac{R_0(1+\alpha_1\theta)R_0(1+\alpha_2\theta)}{R_0(1+\alpha_1\theta)+R_0(1+\alpha_2\theta)}$$

where, $R_{p0} = \frac{R_0R_0}{R_0+R_0} = \frac{R_0}{2}$

$$\therefore \frac{R_0}{2}(1+\alpha_p\theta) = \frac{R_0^2(1+\alpha_1\theta+\alpha_2\theta+\alpha_1\alpha_2\theta^2)}{R_0(2+\alpha_1\theta+\alpha_2\theta)}$$

as α_1 and α_2 are small quantities.

∴ α_1, α_2 is negligible.

So, neglect $\alpha_1, \alpha_2, \theta^2$

or $$\alpha_p = \frac{\alpha_1+\alpha_2}{2+(\alpha_1+\alpha_2)\theta}$$

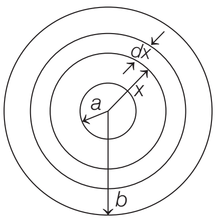
$$= \frac{\alpha_1+\alpha_2}{2}\left[1-\left(\frac{\alpha_1+\alpha_2}{2}\right)\theta\right]$$

[Binomial expansion]

as $(\alpha_1+\alpha_2)^2$ is negligible

$$\therefore \alpha_p = \frac{\alpha_1+\alpha_2}{2}$$

8 Consider a concentric spherical shell of radius x and thickness dx as shown in figure. Its resistance, dR is



$$dR = \frac{\rho\,dx}{4\pi x^2} \qquad \left(\because R = \frac{\rho l}{A}\right)$$

∴Total resistance,

$$R = \frac{\rho}{4\pi}\int_a^b \frac{dx}{x^2}$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

9 As at initial condition the deflection is d while $R_3 = 0$,

then equivalent resistance of R_2 and R_3 $= R_2 + R_3 = R_2 = 30\ \Omega$

Now, when $R_3 = 107\ \Omega$

and $R_2 = 30\ \Omega$

Then, if the deflection is $\frac{d}{2}$, so equivalent resistance should be $\frac{30}{2} = 15\ \Omega$

It is only when equivalent resistance and R_3 and R_g will be parallel to R_2 giving resistance $15\ \Omega$.

Let $R_3 - R_g$ = equivalent $= 30\ \Omega = R$

$$\therefore \frac{1}{R_2} + \frac{1}{R} = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}$$

$$\therefore R_{eq} = 15\ \Omega$$

Thus, R_g must will be $77\ \Omega$ in order to maintain

$$R_3 - R_g = 30$$

$$\Rightarrow 107 - R_g = 30$$

$$= R_g = 77\Omega$$

10 Let $R = at + b$

At $t = 10$ s, $R = 20\ \Omega$

∴ $20 = 10a + b$ ...(i)

At $t = 30a + b$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$a = 1.0\ \Omega/\text{s}$$

and $$b = 10\ \Omega$$

$$\therefore R = (t+10)$$

$$I = \frac{E}{R} = \frac{10}{t+10}$$

$$\Delta q = \int_{10}^{30} I dt$$

$$= \int_{10}^{30}\left(\frac{10}{t+10}\right)dt$$

$$= 10 \log_e (2)$$

11 Effective value of resistance of parallel combination of $20\ \Omega, 30\ \Omega, 60\ \Omega$ is R_1, where

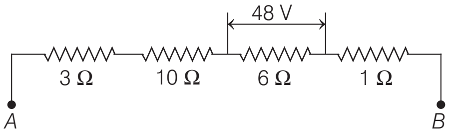
$$\frac{1}{R_1} = \frac{1}{20} + \frac{1}{30} + \frac{1}{60}$$

$$= \frac{3+2+1}{60} = \frac{6}{60} = \frac{1}{10}$$

$$R_1 = 10\ \Omega$$

Similarly, effective value of parallel combination of $24\ \Omega$ and $8\ \Omega$ resistance is given by

$$R_2 = \frac{24\times 8}{24+8} = 6\ \Omega$$



Hence, the circuit may be redrawn as shown in the adjacent figure, where total resistance across A and B,

$$R = 3 + 10 + 6 + 1 = 20\ \Omega.$$

As potential difference across $R_2 (= 6\ \Omega)$ is 48 V, hence

$$V_{AB} = 48 \times \frac{R}{R_2}$$

$$= \frac{48\times 20}{6} = 160\ \text{V}$$

12 $R_1 = \tan\theta = R_0(1+\alpha T_1)$

and $R_2 = \cot\theta = R_0(1+\alpha T_2)$

$\cot\theta - \tan\theta$

$$= R_0(1+\alpha T_2) - R_0(1+\alpha T_1)$$

$$= R_0\alpha(T_2 - T_1)$$

or $$T_2 - T_1 = \frac{1}{\alpha R_0}(\cot\theta - \tan\theta)$$

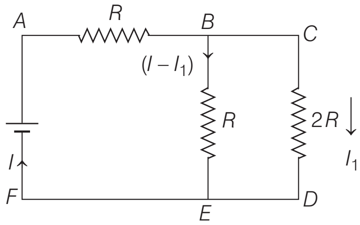
$$= \frac{1}{\alpha R_0}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{2\cos 2\theta}{\alpha R_0 \sin 2\theta}$$

$$= \frac{2}{\alpha R_0}\cot 2\theta$$

$$T_2 - T_1 \propto \cot 2\theta$$

13 $Q = at - bt^2$



$$\therefore I = \frac{dQ}{dt} = a - 2bt$$

when, $t = t_0, I = 0$,

i.e., $a - 2bt = 0$ ...(i)

In loop $BCDEB$,

$I_1(2R) - (I - I_1)R = 0$ or $3I_1 = I$

$$I_1 = \frac{i}{3} = \frac{a-2bt}{3}$$

$$H = \int_0^{t_0}(I_1^2(2h))$$

$$= \frac{2R}{9}\int_0^{t_0}(a-2bt)^2 dt$$

$$= \frac{2R}{9}\left[\int_0^{t_0}(a^2 - 4b^2t^2 - 4dt)dt\right]$$

$$= \frac{2R}{9}\left[\left\{a^2t + \frac{4b^2t^2}{3} - \frac{4bat^2}{2}\right\}_0^{t_0}\right]$$

$$= \frac{2R}{9}\left[a^2t_0 + \frac{4b^2t_0^3}{3} - 2bat_0^2\right]$$

$$t_0 = \frac{a}{2b} \quad \text{[from Eq. (i)]}$$

$$H = \frac{2R}{9}\left[a^2 \times \frac{a}{2b} + \frac{4b^2}{3}\frac{a^3}{8b^3} - 2ab\frac{a^2}{4b^2}\right]$$

$$= \frac{2R}{9}\left[\frac{a^3}{2b} + \frac{a^3}{6b} - \frac{a^3}{2b}\right]$$

$$= \frac{a^3R}{27b}$$